7. ERCOFTAC Symposium on Engineeng Turbulence Modelling and Measurements, ETMM-7, Limassol, Cyprus, 4–6 jun 2008

Wall-Effects on Pressure Fluctuations in Quasi-Incompressible Turbulent Plane Channel Flow

G.A. Gerolymos[1,*], C. Lo[1,†], D. Sénéchal[1,‡], I. Vallet[1,§], B.A. Younis[2,¶]
[1]Université Pierre-et-Marie-Curie, Case 161, 4 place Jussieu, 75005 Paris, France
[2]Department of Civil and Environmental Engineering, University of California, Davis CA 95616, USA

Abstract

The purpose of the present paper is to extend the analysis of turbulent correlations containing pressure fluctuations p', by separating p' not only into rapid $p'_{(\mathrm{r})}$ and slow $p'_{(\mathrm{s})}$ parts (as is usually done in wall-bounded DNS computations), but further into volume (weakly inhomogeneous; $p'_{(\mathrm{r};\mathfrak{V})}$ and $p'_{(\mathrm{s};\mathfrak{V})}$) and surface (strongly inhomogeneous wall-echo; $p'_{(\mathrm{r};w)}$ and $p'_{(\mathrm{s};w)}$) terms. A DNS processing module is developed which computes the above splittings for various correlations containing pressure fluctuations (redistribution, pressure diffusion, velocity/pressure-gradient). Results are presented for DNS computations of quasi-incompressible channel flow, and used for the *a priori* near-wall assessment of existing Reynolds-stress models.

1 Introduction

Starting from the seminal paper of Chou [1], all models [13] for the redistribution tensor ϕ_{ij}, the velocity/pressure-gradient tensor Π_{ij} or the pressure transport $\overline{p'u'_i}$ appearing in the pressure-diffusion tensor $d_{ij}^{(p)}$, are traditionally composed of 4 parts corresponding to the splitting $p' = p'_{(\mathrm{r};\mathfrak{V})} + p'_{(\mathrm{r};w)} + p'_{(\mathrm{s};\mathfrak{V})} + p'_{(\mathrm{s};w)}$. There is at present no possibility to separately measure $p'_{(\mathrm{r};\mathfrak{V})}$, $p'_{(\mathrm{r};w)}$, $p'_{(\mathrm{s};\mathfrak{V})}$, and $p'_{(\mathrm{s};w)}$, and even the simultaneous measurement of p' and $\partial u'_i/\partial x_j$ in the flowfield seems beyond the present measurement technology state-of-the-art. Direct numerical simulation (DNS) offers the possibility to directly compute the different terms [9]. The purpose of the present paper is to revisit the processing of DNS computations to obtain the 5-part decomposition of various p'-related correlations appearing in the Reynolds-stress transport equations.

2 Pressure Fluctuations

The DNS code used [4] solves the compressible Navier-Stokes equations, and in this way computes p' directly, and does not require the solution of a Poisson equation for pressure, as part of the algorithm. This directly computed p' is used to check that the sum of the 5-part-split terms is correct. A processing module is developed, using a Green's function solution for ($p'_{(\mathrm{r})}$, $p'_{(\mathrm{s})}$ and $p'_{(\tau)}$) [9], augmented with free-space Green-function solutions [3] ($p'_{(\mathrm{r};\mathfrak{V})}$, $p'_{(\mathrm{r};\mathfrak{V})}$). The difference between these 2 solutions allows the evaluation of the wall-echo contribution to the pressure-fluctuations. The contribution of each of the 5 components of p' to ϕ_{ij}, Π_{ij} and $\overline{p'u'_i}$ is computed. These results can be used as a database for the analysis and improvement of closures for these terms in the context of Reynolds-stress models (RSMs) [5].

In the limiting case of incompressible constant viscosity flow the Poisson Equation for p' is split into [3]

$$[M \longrightarrow 0\ ;\ \rho,\ \mu \cong \mathrm{const}] \implies$$

$$\nabla^2 p'_{(\tau)} \cong 0 \quad ; \quad \frac{\partial p'_{(\tau)}}{\partial n} = \frac{\partial \tau'_{nj}}{\partial x_j} \cong \bar{\mu}\frac{\partial^2 u'_n}{\partial n^2} \quad \forall \vec{x} \in \partial\mathfrak{V}_w \tag{1}$$

$$\nabla^2 p'_{(\mathrm{BF})} = Q'_{(\mathrm{BF})} \quad ; \quad \frac{\partial p'_{(\mathrm{BF})}}{\partial n} = [\rho f_{\mathrm{V}_n}]' \quad \forall \vec{x} \in \partial\mathfrak{V}_w \tag{2}$$

$$\nabla^2 p'_{(s)} = Q'_{(\mathrm{s})} \quad ; \quad \frac{\partial p'_{(\mathrm{s})}}{\partial n} = 0 \quad \forall \vec{x} \in \partial\mathfrak{V}_w \tag{3}$$

$$\nabla^2 p'_{(r)} = Q'_{(\mathrm{r})} \quad ; \quad \frac{\partial p'_{(\mathrm{r})}}{\partial n} = 0 \quad \forall \vec{x} \in \partial\mathfrak{V}_w \tag{4}$$

*georges.gerolymos@upmc.fr
†lo.celine@etu.upmc.fr
‡dsenecha@ccr.jussieu.fr
§isabelle.vallet@upmc.fr
¶bayounis@ucdavis.edu

The solution of the generic partial differential equation (PDE)

$$\nabla^2 p'_{(m)} = Q'_{(m)} \quad ; \quad y = \pm\tfrac{1}{2}L_y \; : \; \frac{\partial p'_{(m)}}{\partial y} = B'_{(m)\pm}(x,z,t) \tag{5}$$

is split into a volume and a wall-echo part

$$\hat{p}'_{(m)}(\kappa_x,y,\kappa_z,t) = \left\{ \int_{-\frac{1}{2}L_y}^{+\frac{1}{2}L_y} \left[G_{\mathrm{Kim}}(\kappa,y,Y)\,\hat{Q}'_{(m)}(\kappa_x,y,\kappa_z,t)\right] dY \right\} + \hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x,y,\kappa_z,t) \tag{6}$$

$$\hat{p}'_{(m;\mathfrak{V})}(\kappa_x,y,\kappa_z,t) = \left\{ \int_{-\frac{1}{2}L_y}^{+\frac{1}{2}L_y} \left[G_{\infty}(\kappa,y,Y)\,\hat{Q}'_{(m)}(\kappa_x,y,\kappa_z,t)\right] dY \right\} \tag{7}$$

$$\hat{p}'_{(m;w)}(\kappa_x,y,\kappa_z,t) = \hat{p}'_{(m)}(\kappa_x,y,\kappa_z,t) - \hat{p}'_{(m;\mathfrak{V})} \tag{8}$$

where G_{Kim} is the Green's function used by Kim [9] and G_∞ is the free-space Green's function [3] which are solutions of the same ODE but with different boundary-conditions

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] G_{\mathrm{Kim}}(\kappa,y,Y) = \delta(y-Y) \quad ; \quad \left[\frac{dG_{\mathrm{Kim}}}{dy}\right](\kappa, y = \pm\tfrac{1}{2}L_y, Y) = 0 \tag{9}$$

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] \hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x,y,\kappa_z,t) = 0 \quad ; \quad \left[\frac{d}{dy}\hat{p}'_{(m)_{\mathrm{BCs}}}\right](\kappa_x, y = \pm\tfrac{1}{2}L_y, \kappa_z, t) = \hat{B}'_{(m)\pm}(\kappa_x,\kappa_z,t) \tag{10}$$

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] G_{\infty}(\kappa,y,Y) = \delta(y-Y) \quad ; \quad \lim_{|y-Y|\longrightarrow\infty} G_\infty(\kappa,y,Y) = 0 \tag{11}$$

The solution is straightforward for $\kappa \neq 0$

$$G_{\mathrm{Kim}}(\kappa\neq 0,y,Y) = -\frac{\cosh[\kappa(L_y - |y-Y|)] + \cosh[\kappa(y+Y)]}{2\kappa\sinh\kappa L_y} \tag{12}$$

$$\kappa := \sqrt{\kappa_x^2+\kappa_z^2} \neq 0 \; : \; \hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x,y,\kappa_z,t) = \frac{B'_{(m)_+}\cosh[\kappa(\frac{1}{2}L_y+y)] - B'_{(m)_-}\cosh[\kappa(\frac{1}{2}L_y-y)]}{\kappa\sinh\kappa L_y} \tag{13}$$

$$G_\infty(\kappa\neq 0,y,Y) = -\frac{e^{-\kappa|y-Y|}}{2\kappa} \tag{14}$$

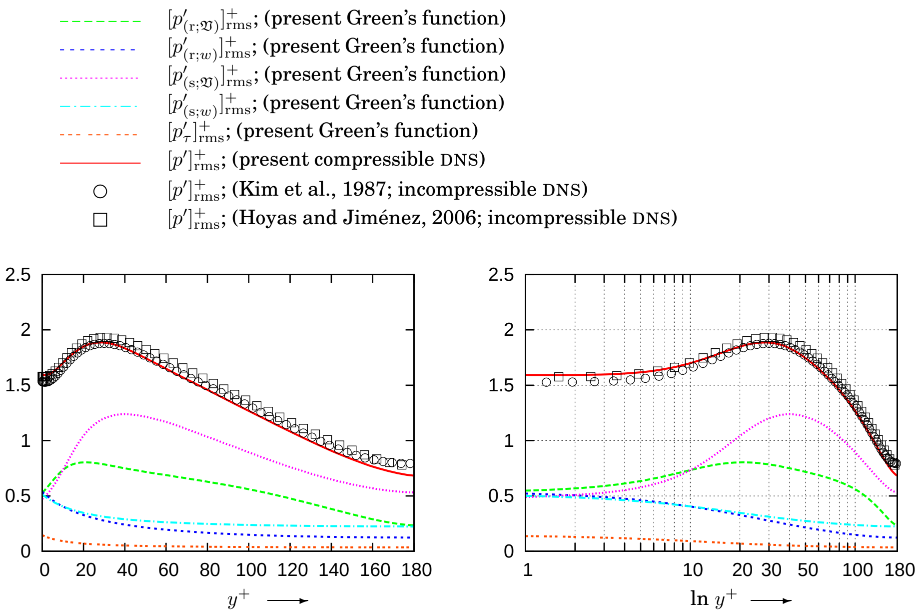

Figure 1: Distributions of rms-values of the 5 terms in the quasi-incompressible p'-splitting, ($p'_{(r;\mathfrak{V})}$, $p'_{(r;w)}$, $p'_{(s;\mathfrak{V})}$, $p'_{(s;w)}$, and $p'_{(\tau)}$, from preliminary results of the present DNS computations ($Re_{\tau_w} = 185$; $M_{\mathrm{B}_w} = 0.3$; $\bar{M}_{\mathrm{CL}} = 0.35$; grid $121\times161\times81$; short observation-time $t^+_{\mathrm{OBS}} = 383$).

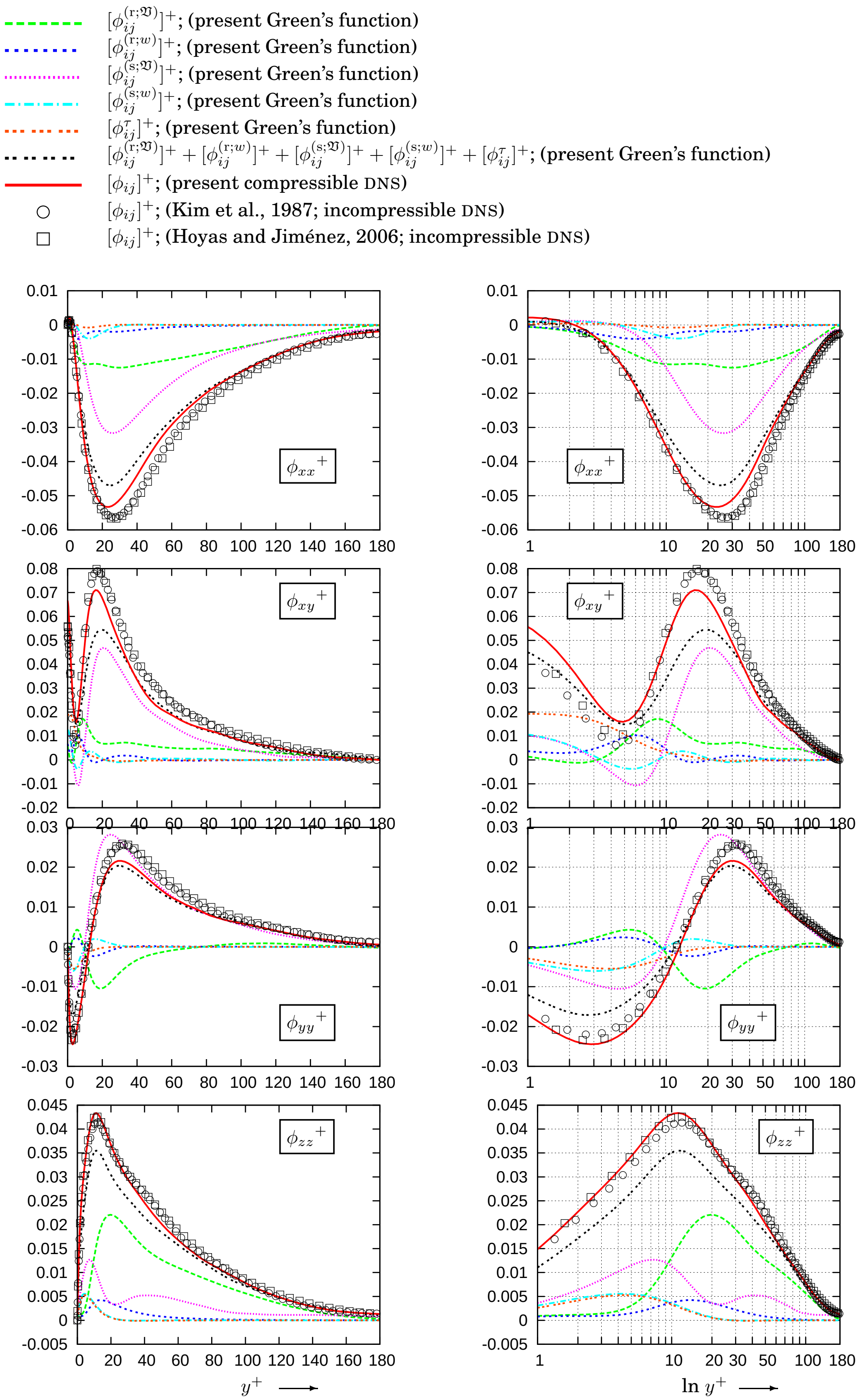

Figure 2: Distributions of ϕ_{ij} for the 5 terms in the quasi-incompressible p'-splitting, ($\phi_{ij}^{(\mathrm{r};\mathfrak{V})}$, $\phi_{ij}^{(\mathrm{r};w)}$, $\phi_{ij}^{(\mathrm{s};\mathfrak{V})}$, $\phi_{ij}^{(\mathrm{s};w)}$, and $\phi_{ij}^{(\tau)}$, from preliminary results of the present DNS computations ($Re_{\tau_w} = 185$; $M_{\mathrm{B}_w} = 0.3$; $\bar{M}_{\mathrm{CL}} = 0.35$; grid $121 \times 161 \times 81$; short observation-time $t^+_{\mathrm{OBS}} = 383$).

but as discussed in Gerolymos-Sénéchal-Vallet [3] for $\kappa = 0$ (corresponding to the instantaneous space-average of p') it is subject to the appropriate compatibility relation [8]. The compatibility relation is simply that the volume-average of each $Q'_{(m)}$, $Q'_{(m;\mathrm{B})}$, is zero, and this is enforced by using $Q'_{(m)} - Q'_{(m;\mathrm{B})}$ as source-term [3]. Notice that this is equivalent to neglecting bulk pressure fluctuations, *ie* to assuming that the volume-average of p', $p'_{\mathrm{B}} = 0 \;\forall t$ [3]. The corresponding solutions are

$$G_{\mathrm{Kim}}(\kappa = 0, y, Y) = G_\infty(\kappa = 0, y, Y) = \frac{|y - Y|}{2} \qquad (15)$$

and

$$\hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x = 0, y, \kappa_z = 0, t) = 0 \qquad (16)$$

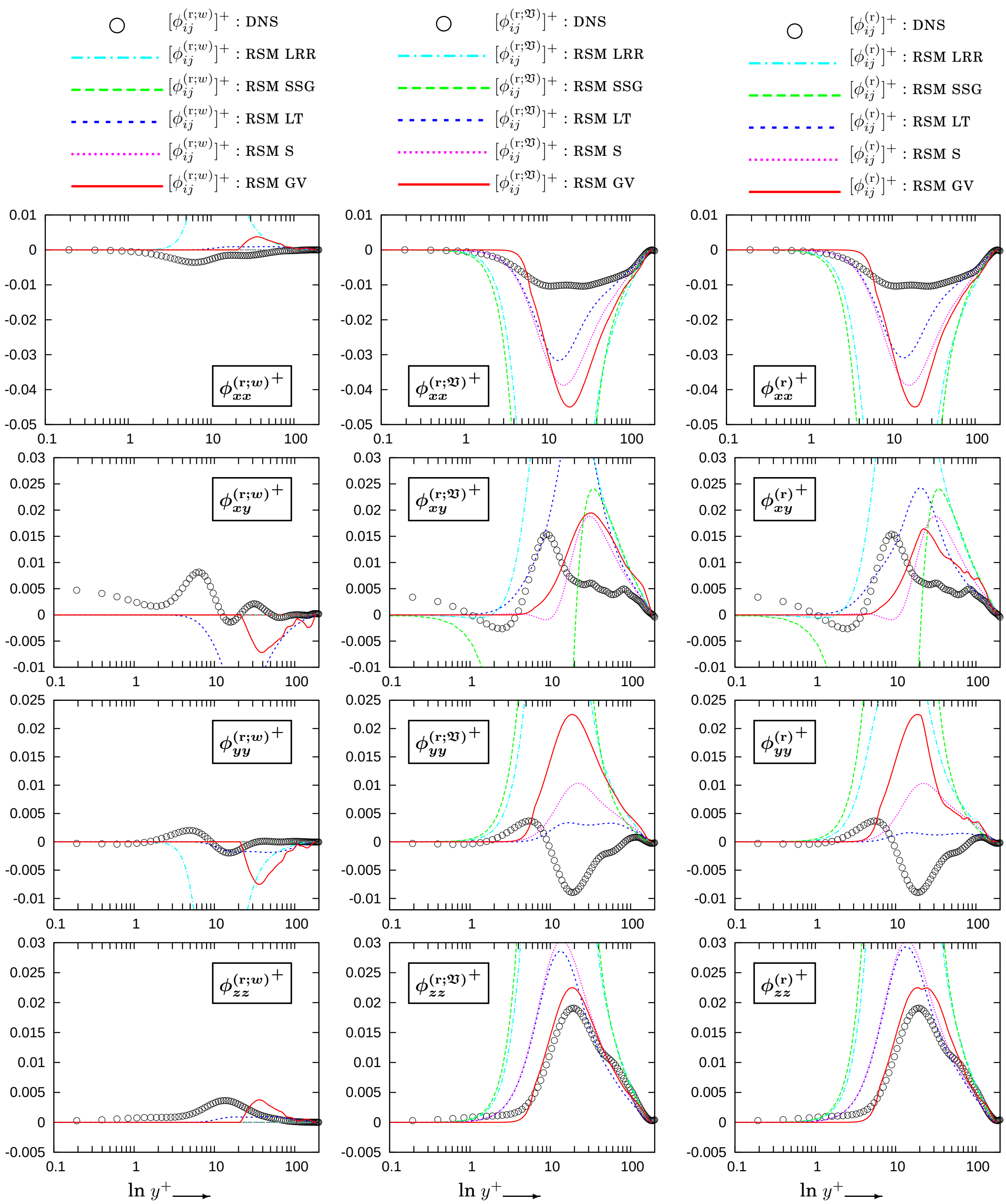

Figure 3: Comparison of *a priori* predictions of $\phi_{ij}^{(r;\mathfrak{V})}$, $\phi_{ij}^{(r;w)}$, and of their sum $\phi_{ij}^{(r)}$, using the second-moment closures of Launder-Reece-Rodi [10] (LRR), Speziale-Sarkar-Gatski [14] (SSG), Launder-Tselepidakis [11] (LT), Suga [15] (S), and Gerolymos-Vallet [6] (GV), with the preliminary results of the present DNS computations ($Re_{\tau_w} = 185$; $M_{B_w} = 0.3$; $\bar{M}_{CL} = 0.35$; grid $121 \times 161 \times 81$; short observation-time $t^+_{OBS} = 383$).

3 DNS Results

The DNS computations were run on an $L_x \times L_y \times L_z = 4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$, using an UW09 scheme with HLLC approximate Riemann solver [4]. The p'_{rms} levels are in very good agreement with the incompressible DNS results of Moser-Lim-Mansour [12] and of Hoyas and Jiménez [7] (Fig. 1). The global p'_{rms} profile, in the present low-Mach-number compressible DNS is obtained directly, without the use of a Green's function approach. Observation of the rms levels of $p'_{(r;\mathfrak{V})}$, $p'_{(r;w)}$, $p'_{(s;\mathfrak{V})}$, $p'_{(s;w)}$, and $p'_{(\tau)}$, computed using the present Green's function approach (Fig. 1), indicates that the levels of the volume terms are very close to the levels of the corresponding wall-echo terms, at the wall, substantiating the importance of wall-blockage. On the othe hand, Stokes-pressure $p'_{(\tau)}$ is substantially lower than the other terms. The overall levels of $p'_{(r)}$, $p'_{(s)}$, and $p'_{(\tau)}$, are in good agreement [3] with the results of Hoyas and Jiménez [7], although

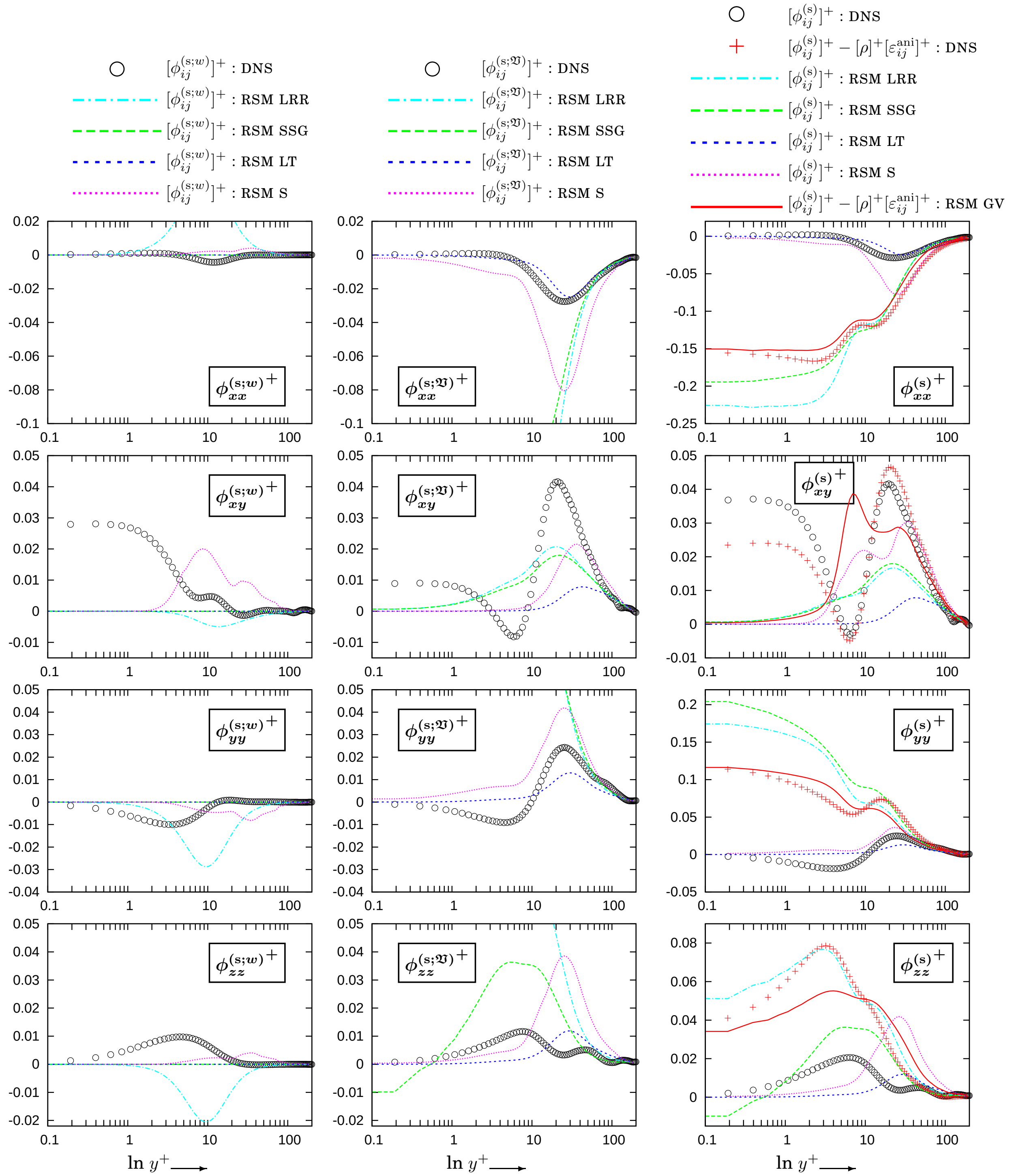

Figure 4: Comparison of *a priori* predictions of $\phi_{ij}^{(s;\mathfrak{V})}$, $\phi_{ij}^{(s;w)} + \phi_{ij}^{(\tau)}$, and of either their sum $\phi_{ij}^{(s)}$, or $\phi_{ij}^{(s)} + \bar{\rho}(\varepsilon_{ij} - \frac{2}{3}\varepsilon\delta_{ij})$ using the second-moment closures of Launder-Reece-Rodi [10] (LRR), Speziale-Sarkar-Gatski [14] (SSG), Launder-Tselepidakis [11] (LT), Suga [15] (S), and Gerolymos-Vallet [6] (GV), with the preliminary results of the present DNS computations ($Re_{\tau_w} = 185$; $M_{\text{B}_w} = 0.3$; $\bar{M}_{\text{CL}} = 0.35$; grid $121 \times 161 \times 81$; short observation-time $t^+_{\text{OBS}} = 383$).

the level of $p'_{(s)}$ is somewhat lower. This is probably due to either insufficient observation time, or insufficient grid resolution for the computation of the source-terms (a C2 scheme was used for the computation of the source-terms).

The originality of the present results lies in the separation between wall-echo and volume terms, as can be observed, *eg* in the 5-terms-splitting of the redistribution tensor ϕ_{ij} (Fig. 2). Although the sum of $\phi_{ij}^{(r;\mathfrak{V})} + \phi_{ij}^{(r;w)} + \phi_{ij}^{(s;\mathfrak{V})} + \phi_{ij}^{(s;w)} + \phi_{ij}^{(\tau)}$ is not exactly equal to the directly computed ϕ_{ij} (insufficient observation time or resolution in the computation the source-terms) the present results ($t^+_{\text{OBS}} = 383$; Fig. 2) are much closer compared to previous results ($t^+_{\text{OBS}} = 82$ [3]), suggesting that future results with $t^+_{\text{OBS}} = 1000$ should be in better agreement. Notice that the same problem has been obesrved by Foysi-Sarkar-Friedrich [2] where various parts of a compressible p' splitting did not sum up exactly to the global computed p'.

The volume terms, $\phi_{ij}^{(\mathrm{r};\mathfrak{V})}$ and $\phi_{ij}^{(\mathrm{s};\mathfrak{V})}$, have values not only of similar level, but also of the same sign, as the corresponding wall-echo terms, $\phi_{ij}^{(\mathrm{r};w)}$ and $\phi_{ij}^{(\mathrm{s};w)}$, respectively (Fig. 2). The Stokes contribution $\phi_{ij}^{(\tau)}$ is in general small in agreement with the observed rms values (Fig. 1). There is however an improtant exception, the shear-stress component ϕ_{xy}, for which $\phi_{xy}^{(\tau)}$ is the largest term near the wall (Fig. 2), where it is higher by a factor of 2 compared to $\phi_{ij}^{(\mathrm{s};\mathfrak{V})}$ and $\phi_{ij}^{(\mathrm{s};w)}$. This is easily explained by the fact that $p'_{(\tau)}$ is generated by the wall-boundary-condition (Eq. 1), which is indeed the normal-to-the-wall derivative of the shear-stress. As a result there is a very good correlation between $p'_{(\tau)}$ and S'_{xy}, at the wall. This suggests that the spurious behaviour of ϕ_{xy}, in the near-wall vicinity, $y^+ < 5$, is due to the contribution of the Stokes pressure.

4 *A priori* Evaluation of SMC

An *a priori* evaluation of various second-moment-closures SMCs, the Launder-Reece-Rodi (LRR) standard linear closure [10], the Speziale-Sarkar-Gatski (SSG) quadratic closure [14], the Launder-Tselepidakis (LT) closure [11], the Suga (S) RSM [15], and the Gerolymos-Vallet (GV) wall-normal-free quasi-linear closure [6]. The present DNS results can be used to separately evaluate various proposals, on a term-by-term basis. Indeed almost all SMCs are composed, both for $\phi_{ij}^{(\mathrm{r})}$ and for $\phi_{ij}^{(\mathrm{s})}$, by 2 parts. The first part, termed either high-Reynolds [10, 14] or quasi-homogeneous [6, 11, 15], can be compared with the volume-term. This is exactly the case in the wall-echo approach [10] but the volume terms are, strictly speaking, weakly inhomogeneous. They are simply the part which does not contain the wall-echo. The second part is exactly the wall-echo term [10]. It certainly also strongly inhomogeneous [6, 11, 15]. In general, although the global prediction of ϕ_{ij} by different models is not unsatisfactory, a term-by-term analysis shows important discrepancies between models and DNS (Figs. 3, 4). The prediction of ϕ_{xy} is unsatisfactory (Figs. 3, 4), all of the models erroneously predicting $[\phi_{xy}]_w = 0$. Observe that the LRR, SSG and GV proposals model $\phi_{ij}^{(\mathrm{s})} + \bar{\rho}(\varepsilon_{ij} - \frac{2}{3}\varepsilon\delta_{ij})$ (Fig. 4).

5 Conclusions

In the present work we extract, form DNS computations, separate information for the volume and the wall-echo terms which compose the pressure-fluctuations in wall turbulence. This technique, which can easily be applied to existing DNS databases, allows a term-by-term evaluation of existing SMCs. It is hoped that this will contribute to a better understanding and improved modelling of wall-turbulence. An initial *a priori* evaluation of existing closures, highlights the generally unsatisfactory term-by-term behaviour.

References

[1] Chou P.Y.: On Velocity Correlations and the Solutions of the Equations of Turbulent Fluctuations, *Quart. Appl. Math.* **3** (1945) 38–54.

[2] Foysi H., Sarkar S., Friedrich R.: Compressibility Effects and Turbulence Scalings in Supersonic Channel Flow, *J. Fluid Mech.* **509** (2004) 207–216.

[3] Gerolymos G.A., Sénéchal D., Vallet I.: Wall-Effects on Pressure Fluctuations in Quasi-Incompressible and Compressible Turbulent Plane Channel Flow, AIAA Paper 2007–3863, 37. AIAA Fluid Dynamics Conference, 25–28 jun 2007, Miami [FL, USA] (2007).

[4] Gerolymos G.A., Sénéchal D., Vallet I.: DNS of Compressible Wall-Turbulence using Low-Diffusion High-Order Upwind Schemes, *Int. J. Num. Meth. Fluids* [submitted; http://www.aerodynamics.fr/publications/node0.php (user: offprints; passwd: turbulence); also AIAA Paper 2007–4196, 18. AIAA Computational Fluid Dynamics Conference, 25–28 jun 2007, Miami [FL, USA]].

[5] Gerolymos G.A., Sénéchal D., Vallet I., Younis B.A.: Comparison of Tensor Representaions of Velocity/Pressure-Gradient, Pressure-Strain and Pressure-Velocity Correlations with Plane Channel Flow DNS Data, 11. European Turbulence Conference, 25–28 jun 2007, Porto [PRT] (2007).

[6] Gerolymos G.A., Vallet I.: Wall-Normal-Free Near-Wall Reynolds-Stress Closure for 3-D Compressible Separated Flows, *AIAA J.* **39** (2001) 1833–1842.

[7] Hoyas S., Jiménez J.: Scaling of the Velocity Fluctuations in Turbulent Channels up to $Re_\tau = 2003$, *Phys. Fluids* **18** (2006) 011702·1–4.

[8] Ince I.L.: *Ordinary Differential Equations* (Dover, New York [NY, USA] 1926), 1. american edn. 254–263, 266–269.

[9] Kim J.: On the Structure of Pressure Fluctuations in Simulated Turbulent Channel-Flow, *J. Fluid Mech.* **205** (1989) 421–451.

[10] Launder B.E., Reece G.J., Rodi W.: Progress in the Development of a Reynolds-Stress Turbulence Closure, *J. Fluid Mech.* **68** (1975) 537–566.

[11] Launder B.E., Tselepidakis D.P.: Directions in 2-Moment Modelling of Near-Wall Turbulence, AIAA Paper 1991–0219 (1991).

[12] Moser R.D., Kim J., Mansour N.N.: Direct Numerical Simulation of Turbulent Channel Flow up to $Re_\tau = 590$, *Phys. Fluids* **11** (1999) 943–945.

[13] Pope S.B.: *Turbulent Flows* (Cambridge University Press, Cambridge [GBR] 2000), ISBN 0–521–59125–2.

[14] Speziale C.G., Sarkar S., Gatski T.B.: Modelling the Pressure-Strain Correlation of Turbulence: An Invariant Dynamical Systems Approach, *J. Fluid Mech.* **227** (1991) 245–272.

[15] Suga K.: Modeling the Rapid Part of the Pressure-Diffusion Process in the Reynolds Stress Transport Equation, *ASME J. Fluids Eng.* **126** (2004) 634–641.